ABSTRACT

In the present paper, we prove that any social choice function satisfies Arrow's principle of Independence of Irrelevant Alternatives (IIA) if individual behavior is menu-dependent. Therefore, Arrow's 'General Possibility Theorem' is not valid when individual preferences are determined by irreducible values. In this context, any aggregation device which satisfies Non-dictatorship and Paretian Unanimity principles (simple majority, for example) also does IIA. This could be an important result for social choice theory, inasmuch as individual behavior determined by irreducible values (self-interest, ideology, ethics, and social norms, for example) can validate representative democracy. The relative importance of such values and the possibility of preference reversals determine the dynamics of social choice according to democratic principles.

KEY WORDS

Constitutional economics; New political economy; Social choice theory; Ideology; Ethics; Values; Irreducible values; Arrow's paradox; Democracy.

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SOCIAL CHOICE AND IRREDUCIBLE VALUES: A POLITICAL ECONOMY APPROACH OF IDEOLOGY[1]

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I. INTRODUCTION

Arrow's 'General Possibility Theorem' establishes that there is no democratic election design that produces transitive and complete social preference relations and also satisfies Non-dictatorship (ND), Paretian Unanimity (PU), and Independence of Irrelevant Alternatives (IIA) principles. Since then, several alternative approaches have been trying to solve this paradox. Buchanan [1954] and Tullock [1967] made important critical comments on Arrow's paradox. They argue that majority voting has some beneficial aspects and that Arrow's theorem does not have much to say about democratic elections schemes. Majority rule is an acceptable one just because it allows logrolling and discussion from which relative unanimity emerges (see, for example, da Silva, 1996). In democracy, competing alternatives could be tested and, if so, replaced by new or old competitive alternatives.

The analysis made by economists of the behavior of people in market place is logically consistent and can be tested. Through empirical testing, one can explain how market and even firms operate when some decisions are made. In public sphere, however, the explanation of the link between private action and collective choice is more difficult because of the existence of some logical problems, as illustrated by Arrow (1963). The collective action theory is accordingly less

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developed than the individual one. Nevertheless, we should explain collective choice, and by explanation we mean (i) the construction of a logically consistent theory and (ii) its empirical testing. Despite Arrow's conclusions (1963), democracy really works and it is a strong empirical evidence which introduces the necessity of alternative approaches to public choice decisions. This is the context of the paper.

The democratic process naturally leads to bargaining, vote bargaining, collusion, and minimal consensus in society. In the democratic process, self-interest, ideology, ethics, and social norms play their role. However, traditional binary choice theory assumes that those values can be reduced to a simple criterion usually associated to the notion of utility. In the present paper, we apply the model of decision where choice is determined by an ordered set of irreducible criteria developed by Moldau (1988, 1992, and 1994) to discuss individual behavior induced by ideology, ethics, and social norms. We call this approach choice by irreducible values model (CIVM).

Contrary to traditional binary choice theory, in which preference relation R is the primitive notion, CIVM departs from the primitive notion of criterion's relative importance. Nonetheless, based on its axioms, it is possible to derive a weak preference relation R (complete and transitive) — Moldau (1994). Garcia (1994) proved an important property of CIVM (Theorem 1): for finite opportunity sets, individual behavior is menu-dependent in the sense of Sen (1993, 1994, and 1995), that is, changes in opportunity set can reverse individual preference over any of two options. This is the key issue to analyze the basic problem of social choice posed by Arrow (1963).

Firstly, we are going to prove (Theorem 2) that any social choice function satisfies the principle of Independence of Irrelevant Alternatives, if individual behavior is menu-dependent. Therefore, Arrow's 'General Possibility Theorem' is not valid when individual preferences are determined by irreducible values. In this context, any aggregation device which satisfies Non-dictatorship and Paretian Unanimity principles (simple majority, for example) also does IIA.

Secondly, we are going to show that this is an important result for social choice theory, inasmuch as individual behavior determined by irreducible values (self-interest, ideology, ethics, and social norms, for example) can validate representative democracy. The relative importance of such values and the possibility of dynamic preference reversals determine the inner nature of social choice according to democratic principles. As noticed by political scientists and thinkers, from Locke to Tocqueville, and also considering Hanna Arent and Antonio Gramsci, democracy implies non-radicalism, persuasion and tolerance, changes in values and ideologies and so forth.

II. IRREDUCIBLE VALUES AND MENU-DEPENDENCE

The standard idea of choice determined by utility maximization presupposes the reducibility of all choice criteria to a single measure of value, usually called utility function. This means that agent's values can be reduced to a single measure of comparison, which enables the establishment of preference relations. Nonetheless, this approach hides some important questions about the structure and the role of individual values when making comparisons, that is, about mechanics of motivational system. In fact, there is no substantive reason to believe that multiple criteria reflecting a spread range of needs, wants, and objectives could be reduced to a single criterion[4]. A neglected treatment of irreducible criteria is widely criticized in literature[5].

[4] Ainslie (1985), for example, suggests a basic opposition between *"visceral satisfactions, closely associated with the consumption of a concrete object … and more subtle satisfactions, such as Knowledge of 'the ideal'…, pursuit of wisdom…"*.
[5] Moldau (1993) summarizes the disagreement about the problem of multiple and irreducible criteria.

Moldau (1988, 1993) presents an alternative method to treat the problem of choice by irreducible criteria, which we call Choice by Irreducible Values Model (CIVM). According to his approach, individual choice is determined by agent's preference ordering over the opportunity set, as it is in most of binary choice theories. The basic difference is that, in his model, agent's preference orderings are determined by the *relative importance* of his set of irreducible criteria. In this section, we present CIVM and deduce their properties.

1. THE MODEL

The problem of choice involves two basic sets: the set of irreducible criteria and the opportunity set. The set of irreducible criteria is referred to as J and is supposed to be comprised of m elements, $m \geq 1$. The opportunity set is referred to as X and is comprised of n elements, $n \geq 1$. Both sets are supposed to be finite.

As state above, the primitive notion in CIVM is the relative importance of the irreducible criteria. Comparisons of any of two options from X are made on the binary relation $\dot{\succeq}$ over the product space JxX of pairs (j,x), where j and x are variables from J and X, respectively. The proposition $(j',x')\dot{\succeq}(j'',x'')$ means that criterion j' at option x' is "at least as important as" criterion j'' at option x''. On the basis of the relative importance relation, we can define relations "more important than" and "as important as".

$$\forall j', j'' \in J \text{ and } \forall x', x'' \in X: \quad (j',x') \dot{\succ} (j'',x'') \Leftrightarrow (j',x')\dot{\succeq}(j'',x'') \wedge \neg(j'',x'')\dot{\succeq}(j',x')$$

$$(j',x') \doteq (j'',x'') \Leftrightarrow (j',x')\dot{\succeq}(j'',x'') \wedge (j'',x'')\dot{\succeq}(j',x').$$

Based on $\dot{\succeq}$ we can define a non-preference relation on X according to criterion j as follows: for any option and for any criterion, x' is "at least as good as" x'' according to criterion *j*, if criterion *j* at x' is as important as it is at x''. Based on this notion, preference and indifference relations according to *j* can be defined as follows:

$$\forall j \in J \text{ and } \forall x', x'' \in X: \quad x' Q_j x'' \Leftrightarrow (j, x'') \dot{\succeq} (j, x')$$

$$x' P_j x'' \Leftrightarrow (j, x'') \dot{\succ} (j, x')$$

$$x' I_j x'' \Leftrightarrow (j, x'') \doteq (j, x').$$

It should be noted that any option is preferred to another if, and only if, criterion *j* at x' is less important than it is at x''. This means that the relative importance of any criterion raises when the necessity behind it is satisfied. Accordingly, Moldau (1993, p.358, fn.) says: "*the preference relation according to a given criterion is defined in terms of an attempted reduction of that criterion's importance*." We can, therefore, read the proposition "criterion j' at option x' is more important than criterion j' at option x''" as follows: at option x', criterion j' is more satisfied than it is at option x''.

Relation $\dot{\succeq}$ is supposed to satisfy the following two axioms:

Axiom 1 (comparability): $\forall j', j'' \in J$ and $\forall x', x'' \in X$, $(j', x') \dot{\succeq} (j'', x'') \vee (j'', x'') \dot{\succeq} (j', x')$.

Axiom 2 (transitivity): $\forall j', j'', j''' \in J$ and $\forall x', x'', x''' \in X$,

$$(j', x') \dot{\succeq} (j'', x'') \wedge (j'', x'') \dot{\succeq} (j''', x''') \Rightarrow (j', x') \dot{\succeq} (j''', x''').$$

Given Axioms 1 and 2, we can say that $\dot{\succeq}$ establishes a weak relative importance ordering and a weak-preference ordering according to *j* at *X*. But it is also possible to establish a weak-importance ordering at *x* on *J*. This is the basic issue to introduce the rule which determines the overall preference relation at *X*. Consider *k(j,x)* as an integer between 1 and *m*, which ranks the criteria in order of importance.

$\forall j', j'' \in J$ and $\forall x' \in X: \quad (j',x') \dot{\succ} (j'',x') \Rightarrow k(j',x') < k(j'',x')$.[6]

This means that if criterion j' is more important than criterion j'' at option x', the ranking number of (j',x') is smaller than the ranking number of (j'',x'); that is, the higher the relative importance of criterion j' at option x' is, the smaller its ranking number is. For the *k*th ranking criterion, the relations of non-preference, preference and indifference can be defined as follows:

$$\forall k \le m \text{ and } \forall x', x'' \in X: \quad x' Q_k x'' \Leftrightarrow (j(k,x'),x') \dot{\succeq} (j(k,x''),x'')$$

$$x' P_k x'' \Leftrightarrow (j(k,x''),x'') \dot{\succ} (j(k,x'),x')$$

$$x' I_k x'' \Leftrightarrow (j(k,x'),x') \doteq (j(k,x''),x'').$$

Finally, we can define the overall preference relation *P* at *X* can be defined as follows:

$$\forall x', x'' \in X: x' P x'' \Leftrightarrow \exists g (g \ge 1 \wedge x' P_k x'') \wedge \forall k (k < g \Rightarrow x' I_k x'').$$

This definition considers that any option x' as preferred to x'' if, and only if, there is some criterion *g*th ranked for which x' is preferred to x'' and, for any other criterion ranked above *g* — that is, any other criterion more important than the *g*th —, x' is indifferent to x''. In other words, the overall preference *P* is determined by the least important criterion for which there is no tie. The least satisfied criterion overcomes those which are more satisfied. The overall indifference relation *I* and the weak preference relation *R* can be defined as follows:

[6] Ties between importance are supposed to be arbitrarily broken. For this issue see Moldau (1993, sec. IV).

$\forall x', x'' \in X : x'Ix'' \Leftrightarrow \forall k (k = 1,\dots,m \Rightarrow x'I_k x'')$.

$$x'Rx'' \Leftrightarrow (x'Ix'' \vee x'Px'')$$

Moldau (1993, p. 359-60) proved that Q_j, Q_k, and R are complete and transitive relations at *X*. Therefore, the individual can order his opportunities from a set of irreducible criteria. Although basic properties of preference ordering are the same as those of any binary choice model, CIVM has a special feature which will be discussed hereafter.

2. MENU-DEPENDENCE

Menu-dependence behavior can be defined as the occurrence of preference reversals[7] when there is some change on the agent's opportunity set. According to Sen (1994), the basic condition for internal consistency of choice fails in such a situation. Suppose that individual *i* prefers option x' to x'', $x', x'' \in X$. Now, assume that we reduce his opportunity set picking up alternative x''' from X and that he says that option x'' is preferred to x'. So, we can say that his preference is menu-dependent, inasmuch changes in menu imply preference reversals.[8]

In this section, we will argue that CIVM does not exclude the possibility of menu-dependence behavior. This proposition was firstly presented in Garcia (1994), and here we only reproduce the general argument of the original proof. If we pay attention to the mathematical structure of the function which determines overall preferences, we can see that it is quite the same as that proposed by Arrow (1963) to social decision functions (also called aggregation devices). Firstly, consider that

[7] This term is usually associated to the failure of independence axiom of expected utility theory — Karni (1987). Notwithstanding, we decided to use it because problems are quite similar.

[8] This behavior is also known as dependence of irrelevant alternatives in social choice theory. According to Arrow (1963), it is a basic requirement for both individual and social rationality. For this issue, see also Mackay (1980) and Sen (1993).

each criterion is an individual of Arrow's system. Therefore, the overall preference would be a kind of social preference determined by individual values.

To study inter-menu problems, we introduce another variable to our analysis: *t*. Menu *t* refers to a specific "situation" in which the individual must establish, based on his values J_t, his preferences P_t at X_t. Note that the set of criteria and the opportunity set are fixed on menu *t*. Accordingly, any of two situations can differ to each order, either because they have different opportunity sets or as a result of differences in criteria sets (changes in individual values, for example). T refers to the set of all possible menus; since *X* and *J* are finite, *T* must also be finite. X_T refers to the set of all $X_{t'}$, on a basis that $t' \in T$.

Given axioms 1 and 2, we know that the weak preference relation according to *j* is a complete and transitive non-preference relation at *X*. This proposition including menu specification can be restated as follows:

$\forall t \in T$, $\forall j \in J$, and $\forall x', x'', x''' \in X$: $x' Q_{jt} x'' \vee x'' Q_{jt} x'$

$$x' Q_{jt} x'' \wedge x'' Q_{jt} x''' \Rightarrow x' Q_{jt} x'''$$

Referring to the weak preference according to *j* in situation *t* as R_{jt} and the weak overall preference in situation *t* as R_t, we can deduce Lemma 1.

Lemma 1 **-** *Given Axioms 1 and 2, for any situation* $t \in T$, *and for any of two options* $x', x'' \in X_t$, *if* $\forall j (j \in J_t \Rightarrow x' R_{jt} x'')$, *then* $x' R_t x''$.

Proof: Suppose that it is was not the case. Then, there would be a situation $t \in T$ and two options $x', x'' \in X_t$, *on a basis that* $\forall j (j \in J_t \Rightarrow x' R_{jt} x'')$, *but* $x'' P_t x'$. *We know that if* $\forall j (j \in J_t \Rightarrow x' R_{jt} x'')$, *then for any of two criteria it would be* $j', j'' \in J_t$, $(j'', x'') \dot{\succeq} (j'', x')$. *Consider* j' *as the decisive criterion at* x'. *In this case, we know that* $(j'', x') \dot{\succeq} (j', x')$ *and, according to Axiom 2, that* $(j'', x'') \dot{\succeq} (j', x')$. *Nonetheless, if* x'' *were* *preferred to* x'*, then there would be two decisive criteria, ie* $j', j'' \in J_t$ *of ranking number k on a basis that* $(j',x') \dot{\succ} (j'',x'')$*, which constitutes a contradiction. Therefore, we can conclude that for any situation* $t \in T$*, and for any of two options* $x', x'' \in X_t$, if $\forall j(j \in J_t \Rightarrow x'R_{jt}x'')$*, then* $x'R_t x''$.

Lemma 1 states that CIVM satisfies the well-known *weak Paretian Unanimity* condition imposed on Arrow's system. Now, presuppose that there is no criterion which, for any situation, is decisive in determining the preference relation between any of two alternatives. This is to say: assume the condition of *Non-dictatorship* (Arrow, 1963). In our approach, this assumption implies that we explicitly exclude the possibility of lexicographic criteria in determining individual's preference. Therefore, any criterion can be satisfied and, this being the case, its relative importance is lowered. The possibility of lexicographic criteria is not excluded in CIVM, therefore we postulate Axiom 3, which is called non-dominance condition.

Axiom 3: $\neg\exists j \in J$, on a basis that $\forall t \in T$ and $\forall x', x'' \in X_t$, $x'R_{jt}x'' \Rightarrow x'R_t x''$.

Now, we can define the menu-dependence condition as follows: any function which determines overall preference based on *J* is considered menu-dependent if, and only if, there are two situations $t', t'' \in T$ and two options $x', x'' \in X_T$ on a basis that criteria sets are the same in both situation $(J_{t'} = J_{t''})$, but $x'R_{t'}x'' \wedge \neg(x'R_{t''}x'')$. Therefore, menu-dependence is the negation of *independence of irrelevant alternatives*.

Finally, we can postulate Theorem 1:

Theorem 1 - *Given Axioms 1, 2, and 3, individual behavior is menu-dependent.*

Proof: The same argument as that used to prove Arrow's 'General Possibility Theorem'.[9]

[9] For this issue see Arrow (1963), Mackay (1980) and Garcia (1994).

This is a very intuitive result: since comparisons are established according to the relative importance of the *k*th criteria at x' and at x'', the preference relation between any of two options does not depend solely on x' and x''. Therefore, menu-dependence behavior emerges as a result of choice determined by a set of irreducible values, none of them prevailing over all other criteria in any possible situation.

III. SOCIAL CHOICE AND IRREDUCIBLE VALUES

The basic property of menu-dependence of individual preference has important effects on social choice. In this section, we prove that, if Axioms 1 to 3 are held for all individuals, then for any social decision function *r*, *r* satisfies the principle of *Independence of Irrelevant Alternatives* (Arrow, 1963). Following, we analyze simple majority rule under conditions of irreducible and non-dominant individual values.

1. INDEPENDENCE OF IRRELEVANT ALTERNATIVES

Consider *H* as the set of all individuals *i*, in which *h*≥1 elements are comprised. We can say that, if Axioms 1, 2 and 3 are held for all individuals *i*, their behavior is menu-dependent. That is to say, if the choice of any individual is determined by a set of irreducible and non-dominant criteria, then menu-dependence is held for all individuals in the society.

Axiom 4: $\forall i \in H$, individual behavior satisfies Axioms 1, 2, and 3.

Given Axiom 4, we can say that $\forall i \in H$, and that there are two situations $t', t'' \in T$ and two options $x', x'' \in X_{t'}, X_{t''}$ on a basis that $x' R_{t'} x'' \wedge \neg(x' R_{t''} x'')$. Similarly, we can say that if Axioms 1 to 4 are held, the following proposition also is:

Proposition 1: $\forall i$, if $\forall t',t'' \in T$ and $\forall x',x'' \in X_T$, $x'R_{it'}x'' \Rightarrow x'R_{it''}x''$, then $i \notin H$.

In social choice theory, Independence of Irrelevant Alternatives is defined as follows: $\forall t',t'' \in T$ and $\forall x',x'' \in X_T$, if $\forall i \in H$, $x'R_{it'}x'' \Rightarrow x'R_{it''}x''$, then $x'S_{t'}x'' \Rightarrow x'S_{t''}x''$, where S_t refers to social weak-preference relation over any of two options and is supposed to be complete and transitive. For our purpose, we define IIA by its negative form:

$\forall t',t'' \in T$, $\forall x',x'' \in X_T$, if $x'S_{t'}x'' \wedge \neg(x'S_{t''}x'')$, then $\exists i \in H$, $x'R_{it'}x'' \wedge \neg(x'R_{it''}x'')$.

Now, we can state the reasonable condition for IIA. For such a purpose, it should be noted that, if for any of two situations and any of two options there is an individual whose preference is menu-dependent, then we can say[10] that if $x'S_{t'}x'' \wedge \neg(x'S_{t''}x'')$, then $\exists i \in H$, $x'R_{it'}x'' \wedge \neg(x'R_{it''}x'')$, $\forall t',t'' \in T$, $\forall x',x'' \in X_T$. Therefore, we only need to ensure that $\forall t',t'' \in T$, $\forall x',x'' \in X_T$, $\exists i \in H$, $x'R_{it'}x'' \wedge \neg(x'R_{it''}x'')$, to prove that IIA is satisfied. To facilitate our analysis, proposition 2 will describe the negation of the necessary condition for IIA.

Proposition 2: $\exists t',t'' \in T$, $\exists x',x'' \in X_T$, on a basis that $\forall i \in H$, $x'R_{it'}x'' \Rightarrow x'R_{it''}x''$.

Following, Lemma 2 proves that proposition 1 is inconsistent with the negation of the necessary condition for IIA (proposition 2), which means that proposition 1 is a necessary condition for IIA.

Lemma 2 - *Proposition 1 implies the necessary condition for IIA.*

Proof: Suppose that it was not the case. Therefore, we have that propositions 1 and 2 are simultaneously held as follows:

[10] This logical argument can be described as follows: if A is true, then B⇒A is also true. Therefore, any proposition B, either true or false, can imply any true proposition A.

$\forall i$, *if* $\forall t',t'' \in T$ *and* $\forall x',x'' \in X_T$, $x'R_{it'}x'' \Rightarrow x'R_{it''}x''$, *then* $i \notin H$ *(proposition 1) and*

$\exists t',t'' \in T$, $\exists x',x'' \in X_T$, *on a basis that* $\forall i \in H$, $x'R_{it'}x'' \Rightarrow x'R_{it''}x''$ *(proposition 2).*

In proposition 2, we have that $\forall i,\ i \in H \Rightarrow (x'R_{it'}x'' \Rightarrow x'R_{it''}x'')$ *and, in proposition 1, that* $\forall i,\ (x'R_{it'}x'' \Rightarrow x'R_{it''}x'') \Rightarrow i \notin H$. *Therefore, we have that* $\forall i,\ i \in H \Rightarrow i \notin H$. *Analyzing this proposition, we can conclude that* $\neg\exists i, i \in H$, *otherwise there would be a contradiction on a basis that* $i' \in H \wedge i' \notin H$. *Therefore, H is necessarily an empty set. Nonetheless, this proposition contradicts the premise that the number of elements from H is larger than 1, h≥1. Therefore, we can conclude that proposition 1 implies the necessary condition for IIA.*

Theorem 2 *- Given Axioms 1 to 4, for any social decision function r, r satisfies the principle of Independence of Irrelevant Alternatives.*

Proof: It is reasonable to consider that Axioms 1 to 4 imply proposition 1, which implies the necessary condition for IIA, according to Lemma 2.

2. SIMPLE MAJORITY

Given Theorem 2, we can inquire whether Arrow's 'General Possibility Theorem' remains valid in a context of individual choice guided by irreducible values. In this section, we analyze a democratic election design based on simple majority which satisfies principles of Unrestricted Scope (US), Paretian Unanimity (PU), and Non-dictatorship (ND). Prior thereto, we will introduce some definitions and premises about the voters' behavior and majority rules.

Assume that any voter *i* satisfies Axioms 1 to 3. For any election *t*, we assume that, for any voter i, his vote for any candidate $x \in X_t$ is a function of his individual preferences over X_t. $V_{i,t}(x)$ refers to the value *i*'s vote at *x* in election *t*, on a basis that $V_{i,t}(x)=1$ if i votes for candidate x, and $V_{i,t}(x)=0$ if i does not vote for x. Now, we introduce the basic relation between voters' preferences and their votes: $\forall i \in H$, $\forall t \in T$, and $\forall x', x'' \in X_t$, $V_{i,t}(x')=1 \Leftrightarrow x' P_{it} x''$. As a consequence, we have that, if any voter has more than one candidate in his most preferred equivalence class, then for any candidate $V_{i,t}(x)=0$.

The overall value of any candidate x, referred to as $V_t(x)$, is the summation of $V_{i,t}(x)$, $i=1,\ldots,h$. That is: $V_t(x)=\sum_{i=1}^{h} V_{it}(x)$. For any candidate x, $h \geq V_t(x) \geq 0$. Given individual votes in any election, we can define relations of social weak-preference (*S*), social preference (*SP*), and social indifference (*SI*). $\forall i \in H$, $\forall t \in T$, and $\forall x', x'' \in X_t$:

$x' S_t x'' \Leftrightarrow V_t(x') \geq V_t(x'')$,

$x' SP_t x'' \Leftrightarrow V_t(x') > V_t(x'')$, and

$x' SI_t x'' \Leftrightarrow V_t(x') = V_t(x'')$.

Given this democratic election design, we can prove that S_t is a complete and transitive relation at X_t. In the other hand, this social decision function satisfies Paretian Unanimity and Non-dictatorship. The proofs for Lemmas 3 to 6 are presented in Appendix 1. Inasmuch any social decision function which satisfies Axioms 1 to 4 also does Independence of Irrelevant Alternatives, we can say that this particular election design is an Arrowian social choice.

Theorem 3 - *Given Axioms 1 to 4, the simple majority rule described above satisfies IIA, US, UP, and ND conditions*.

Proof: Directly from Theorem 2 and Lemmas 3 to 6.

Following is a very simple example of how this election design can be applied. Suppose an election among three candidates x', x'', and x''' based on individuals' preferences of i', i'', and i'''. Figure 1 shows voters' preferences. In this situation, each candidate is voted once: i' votes for x', i'' votes for x''', and i''' votes for x''. Therefore, we can say that x' is socially indifferent to x'', which is also indifferent to x'''.

Figure 1 - Example of an election

	i'	$x'Px''$	$x''Px'''$
voters	i''	$x'''Px'$	$x'Px''$
	i'''	$x''Px'''$	$x'''Px'$

This example is also useful to illustrate the intuitive idea implicit in the choice by irreducible values and the menu-dependence behavior. Suppose that, whatever the reason, candidate x''' is excluded from the opportunity set. In this case, we have a new menu on a basis that, if individual preferences were not changed, we would have the election of x', that is, $x'SPx''$. This example would illustrate the possibility of social menu-dependence. Nonetheless, this argument cannot be applied, because when we exclude x''' from the opportunity set, the menu-dependent behavior of voters can change individual preferences and votes.[11]

Finally, we can imagine situations in which ties are not accepted — elections for public offices, for example. In fact, the democratic election design described above

[11]Arrow (1963) argues that this social behavior would not be rational. Here, we propose an alternative approach to this problem. In our analysis, this behavior cannot occur since the exclusion of any candidate changes voters' preferences over the other candidates.

admits the possibility of two candidates being equally voted. If they are the most preferred candidates, but only one winner is to arise from the election, there would be an indetermination. In such a case, we could figure out a further step to elections: the two most voted candidates would play in a run-off. This procedure could generate only one winner.[12]

IV. FINAL COMMENTS

The first important conclusion in this paper is that tolerance and persuasion are the bases of democratic normative framework. Based on Axiom 3, we can perceive the nature of democracy as a non-dominant individual value schema or an Institutional set which demands tolerance.

This conclusion naturally leads to another important comment from the results presented herein. The formation of a coalition can result that one single winner emerges. A coalition corresponds to an agreement entered into by two or more players to coordinate their actions on a basis that a more advantageous outcome prevails to the members of the coalition as compared with that prevailing from an non-cooperative action. Despite free riding, as proposed initially by Olson (1965), the logic of collective action implies that, if transaction costs involved in a coalition formation are low or nil, individual agents improve private well-being by implementing collective actions. Accordingly, if a coalition prefers *x'* to *x* , it means that every member prefers *x'* to *x*. Therefore, a coalition can be modeled as an individual. In society, where many coalitions exist, a majority can be attained. This result can emerge from a dynamic ideological bargaining. If the transaction involved in the process is costless, the process could, and we here are just suggesting, select Condorcet winners as final outcomes.

[12] This procedure assures the occurrence of a Condorcet winner.

The last important conclusion leads to some reflection upon the role of electoral researches made by newspapers and independent institutions. The feasible outcomes set *X* is altered when one candidate is said to be “out of the play”. The electoral pools researches can alter the information set and the agents’ preference orderings, without any change in individual values. This is an important intuitive result derived from the theory proposed above. The information set can alter the menu perceived by agents and, if so, it becomes a very important instrument for the decisions to be made following a collective choice.

The main conclusion is that democracy requires a normative rule associated with tolerance, bargain and, translating into economics jargon, dynamic preference orderings. The existence of radicals guided by lexical values would fail to meet the condition of non-dominant values posed by Axiom 3. Accordingly, democracy is incompatible with antiestablishmentarian behavior. This is the core of democratic collective choice process.

V. APPENDIX

Lemma 3 - *Given Axioms 1 to 4, the simple majority rule determines a complete social weak-preference relation at* X_t *for any election t.*

Proof: Suppose that it was not the case. Therefore, there would be an election t' *with two options* x' *and* x''*, on a basis that* $\neg(x' S_{t'} x'')$ *and* $\neg(x'' S_{t'} x')$*. In this case, the definition of social weak-preference would imply that*: $\neg(V_{t'}(x') \geq V_{t'}(x''))$ *and* $\neg(V_{t'}(x'') \geq V_{t'}(x'))$*. This would also mean that* $V_{t'}(x') > V_{t'}(x'')$ *and* $V_{t'}(x'') > V_{t'}(x')$*, which would constitute a contradiction. Therefore, for any election* t *and any of two candidates* x' *and* x''*,* $x' S_t x''$ *or* $x'' S_t x'$*. Therefore, we have that* S_t *is a complete binary relation on* X_t*.*

Lemma 4 *- Given Axioms 1 to 4, the simple majority rule determines a transitive social weak-preference relation at* X_t *for any election t.*

Proof: Suppose that it was not the case. Therefore, there would be an election t' *with three options* x', x'', *and* x''' *on a basis that* $x' S_{t'} x''$ *and* $x'' S_{t'} x'''$, *but* $\neg(x' S_{t'} x''')$. *In this case, the definition of social weak-preference would imply that:* $V_{t'}(x') \geq V_{t'}(x'')$ *and* $V_{t'}(x'') \geq V_{t'}(x''')$, *but* $\neg(V_{t'}(x') \geq V_{t'}(x'''))$. *Nonetheless, if* $V_{t'}(x') \geq V_{t'}(x'')$ *and* $V_{t'}(x'') \geq V_{t'}(x''')$, *we would have that* $V_{t'}(x') \geq V_{t'}(x''')$, *which would constitute a contradiction. Therefore, for any election* t *and any candidates* x', x'', *and* x''', *if* $x' S_t x''$ *and* $x'' S_t x'''$, *then* $x' S_t x'''$. *Therefore, we have that* S_t *is a transitive binary relation at* X_t.

Lemma 5 *- Given Axioms 1 to 4, we have that simple majority rule satisfies Paretian Unanimity principle.*

Proof: Suppose that it was not the case. Therefore, there would be an election t' *with two options* x' *and* x'', *on a basis that* $\forall i \in H(x' R_{it'} x'')$, *but* $\neg(x' S_{t'} x'')$. *In this case, the definition of social weak-preference would imply that* $V_{t'}(x'') > V_{t'}(x')$, *because* $\neg(V_{t'}(x') \geq V_{t'}(x''))$. *Nonetheless, if it were the case, we would have that* $\exists i \in H(x'' P_{it'} x')$. *This would contradict the statement that* $\forall i \in H(x' R_{it'} x'')$. *Therefore, for any election* t *and any of two candidates* x' *and* x'', *if* $\forall i \in H(x' R_{it'} x'')$, *then* $x' S_{t'} x''$. *Therefore, we have that simple majority rule satisfies Paretian Unanimity principle.*

Lemma 6 *- Given Axioms 1 to 4, we have that simple majority rule satisfies Non-dictatorship principle.*

Proof: Directly from the definition of simple majority rule.

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